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ST. LAWRENCE HIGH SCHOOL

FOR GOD AND COUNTRY

SELECTION TEST

Subject:Statistics **Class: XII** **F. M. 70**

Solution **Date: 23.11.2019**

PART : A

Q1. Answer the following questions. **2X4=8**

i. When does a discrete random variable follow Poisson distribution?

Ans. For bernoullian variable when n is large and p is so small and np is constant.

ii. Write a short note on Percentile scale.

Ans. The basic assumptions are the distribution is rectangular and percentile successive differences are equal. To determine the scale value of a raw score one has to find the percentile position of that score.

iii. What is Critical region?

Ans. The region where if test statistic lies we reject the null hypothesis.

OR What do you mean by power of test ?

Ans. Prob{ Reject null hypothesis given it is true}

iv. Write the components of time series data.

Ans. Secular trend, seasonal trend, cyclical trend and eregular fluctuation.

OR

Write the normal equations in exponential trend fitting.

Ans. $\sum_{i=1}^{n} y_i = na + b \sum_{i=1}^{n} t_i$ and $\sum_{i=1}^{n} t_i y_i = a\sum_{i=1}^{n} t_i + b \sum_{i=1}^{n} {t_i}^2$

Where y_i= ln Y_i

Q2. Answer the following questions. **3X8=24**

i. Find and unbiased estimator of square of population mean where population units follow $N(\mu , 1)$

Ans. A random sample is drawn and mean is m(say). $E(m^2 - \frac{1}{n}) = \mu^2$.

ii. Determine f(x), the p.m.f., from $f(x) = \frac{\lambda}{x} f(x-1), x = 1,2,3,..,$ where f(x) is non-zero for non-negative integral values of the random variable x. Find also the probability that X is greater than zero.

Ans. $f(x) = \frac{\lambda^x}{x!} F(0), \sum f(x) = 1 \Rightarrow f(0) = e^{-\lambda}$. So $f(x) = e^{-\lambda} \frac{\lambda^x}{x!}$.

iii. Derive the expression of harmonic mean of a random variable X ≈ Binomial(n ,p).

Ans. $H = E(\bar{x} + x - \overline{x})^{-1} \Rightarrow H = 1 - \frac{s^2}{\bar{x}^2}$

OR Find the expectation of number of throws required to get the third '6' in repeated throw of an unbiased die.

Ans. X: no of throws

$E(X) = (\frac{1}{6})^3 \left(1 + \sum(\frac{5}{6})^{x-3} \left(x - 1_{c_2}\right)\right)$

iv. The wage distribution of 100 workers are given as mean wage Rs 500 and standard deviation Rs 50. Find the maximum wage of 10 highest paid workers. Given $\int_0^x \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = 0.4$ for x= 1.286.

Ans. Let Rs a be the maximum wage of 10 highest paid workers.

P(x>a)=0.1 $\Rightarrow$ P(Z < $\frac{a-500}{50}$) = P(Z < 1.286) $\Rightarrow a = 500 + 50 * 1.286 \Rightarrow a = 564.30$

OR Derive mean deviation about mean of normal distribution.

Ans. MD $\mu = \int_{-\infty}^{\infty} \frac{Ix-\mu I}{\sigma} \frac{1}{\sigma\sqrt{2\pi}} e^{(\frac{x-\mu}{\sigma})^2} dx = \sigma\sqrt{\frac{2}{\pi}}$

v. Write a short note on moving average method used in time series.

Ans. By moving average method we determine the trend values of time series data.For n yearly moving average method take the average of observations of n years starting from 1^{st}, 2^{nd} .... Years and put them in the middle year.When n is even use centered average. By thi method we can not forecast.

OR Write a short note on chi-square test.

Ans. Frequency or Pearsonian $\chi^2 = \frac{(f_o - f_e)^2}{f_e}$

Uses: Test for Goodness of fit and test for independence.

vi. Derive the expression of coefficient of determination from regression equation y on x.

Ans. V(Y) = $\frac{1}{n}\sum_{i=1}^{n}(Y_i - \bar{Y})^2 = \frac{1}{n}\sum_{i=1}^{n}(\bar{y} + r\frac{s_y}{s_x}(x_i - \bar{x}) - \bar{y})^2 = r^2 s_y^2$

r^2 is known as coefficient of determination.

vii Write a short note on random number series.

Ans. A series of numbers expressed in rows and columns in blocks of 4. The characteristic of these numbers is such that all the single digit number from 0 to 9 occur same number of time, ie, probability of selecting any number is $\frac{1}{10}$. The numbers 00, 01, ..., 99 occur same number of time with probability of each $\frac{1}{100}$. For three digit numbers the probability is $\frac{1}{1000}$ and so on. Some commonly used random number series are Tippett's series, Fisher-Yate series, ISI series etc.

viii. Write the control limits of number defective chart for both the cases when standards are given and not given.

Ans. E(np) =nP and $\sigma_{np=\sqrt{nP(1-P)}}$

Case 1 : Standard given P= p'

LCL= p' – 3 $\sqrt{p'(1-p')}$, CL = p' and UCL= p' + 3 $\sqrt{p'(1-p')}$

Case 2 : Standard not given P= $\bar{p}$

LCL= $\bar{p} - 3\sqrt{\bar{p}(1-\bar{p})}$, CL = $\bar{p}$ and UCL = $\bar{p} + 3\sqrt{\bar{p}(1-\bar{p})}$

OR Write the control limits of range chart for both the cases when standards are given and not given.

Ans. E(R) =$d_2\,\sigma$ and $\sigma_{R=D\sigma}$

Case 1 : Standard given $\sigma = \sigma'$

LCL =$d_2\,\sigma'$ - 3D σ' = $D_1\,\sigma'$, CL = $d_2\,\sigma'$ and UCL= $d_2\,\sigma'$ + 3D σ'.

Case 2 : Standard not given $\sigma = \frac{\bar{R}}{d_2}$

LCL = $\bar{R}$ - 3D $\frac{\bar{R}}{d_2}$ = $D_3\bar{R}$, CL =$\bar{R}$ and UCL = $\bar{R}$ + 3D $\frac{\bar{R}}{d_2}$ = $D_4\bar{R}$

Q3. Answer the following questions. **5X4=20**

i. Give the theory behind control chart technique.

Ans. write the points.. definition, rational sub group formation, selection of statistic, for non normal hebyshev's inequality P(μ-3σ < T < μ+3σ) > $\frac{8}{9}$ and for normal P(μ-3σ < T < μ+3σ) = 0.9973

ii. Derive the expression of standard error of sample proportion in SRSWR.

Ans. Here x_i *and* x_j *are independent*. E(x_i) $= \mu$, v(x_i) $= \sigma$ and cov (x_i, x_j)=0. V($\bar{x}$)= $\frac{\sigma}{\sqrt{n}}$. Then replace *by* $\sqrt{p(1-p)}$.

OR Derive the expression of standard error of sample proportion in SRSWOR.

Ans. Here x_i *and* x_j *are independent*. E(x_i) $= \mu$, v(x_i) $= \sigma$ and cov (x_i, x_j)=$\frac{\sigma^2}{N-1}$. V($\bar{x}$)= $\sqrt{\frac{N-n}{N-1}}\ \frac{\sigma}{\sqrt{n}}$. Then replace *by* $\sqrt{p(1-p)}$.

iii. Derive the expression of rth order central moment of N(μ ,σ^2)

Ans. $\mu_{2r+1} = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}(\ x-\ \mu)^{2r+1}e^{\frac{-1}{2\sigma^2}(x-\mu)^2}dx$= 0 since odd function

. $\mu_{2r} = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}(\ x-\ \mu)^{2r}e^{\frac{-1}{2\sigma^2}(x-\mu)^2}dx$

= .$\frac{2}{\sigma\sqrt{2\pi}}\int_{-0}^{\infty}(\ x-\ \mu)^{2r}e^{\frac{-1}{2\sigma^2}(x-\mu)^2}dx$

substituting t = $\frac{(x-\mu)}{\sigma}$ and u = $t^2/2$

= (2r-1) **(2r-3)*..***3*1 σ^{2r}

OR Derive the expression of regression equation of y on x.

Ans. In a scatter diagram let ith plotted point be(x_i, y_i) and the predicted best fitted line be Y=a+bX . e_i = (y_i-Y_i) = (y_i- $a - bx_i$)

Sum of square of errors E = $(y_i - a - bx_i)^2$

Minimizing E by differentiating partially w.r.t.a and b we get the normal equations

$\sum_{i=1}^{n} y_i = na + b\ \sum_{i=1}^{n} x_i$ and $\sum_{i=1}^{n} x_i y_i = a\sum_{i=1}^{n} x_i + b\ \sum_{i=1}^{n} {x_i}^2$

Solving we get, $\hat{b}$ = $\frac{cov(x,y)}{V(x)}$ = b_{xy} . Hence the regression equation is y - $\bar{y}$ = b_{xy}(x- $\bar{x}$).

iv. Among 8 dice, m number of dice are so biased that those always show six as face value. The null hypothesis H_0 : m=2 against H_1: m= 1 is rejected if less than three six appear when all those eight are being thrown. Find probability of type I error and power of test.

Ans. X follows Bin (8-m ,$\frac{1}{6}$).

To test H_0 : m=2 against H_1: m=1.

P(type I error) = P(Reject H_0 given m=2)=$(\frac{5}{6})^6$

P(type II error) = P(Accept H_0 given m=1) = 1 – P(X < 3) = $(\frac{5}{6})^7 + 7_{c_1}\ (\frac{5}{6})^6\ \left(\frac{1}{6}\right)$

=2 $(\frac{5}{6})^6$

PART B

Q1. Select the correct alternatives. **1x10**

i. The correlation coefficient between religion and income is

a. 1 b. -1 c. 0 **d. none of these**

ii. The unbiased estimator of population mean is

a. sample mean b. sample median c. sample median d. none of these

iii. The maximum probability of type II error is

a. level of significance b. power of test c. Type I error **d. none of these**

iv. Response bias is

a. sampling bias **b. non sampling bias** c. response bias d. none of these

v. A random variable follows N(25 , 9). Then P(X = 17) is

a.0 b. 1 c. 0.5 d. none of these

vi. Which of the following control chart is used for attribute

a. mean chart b. range chart **c. fraction defective** d. none of these

vii. The standard deviation of a Poisson distribution is 2, its b_1 is equal to

a $\frac{1}{2}$ b $\frac{1}{4}$ c 1 **d. none of these**

viii. Given two points on scatter diagram the correlation coefficient between x and y is

a. +1 b. -1 c. 0 **d. a or b**

ix. Pdf is the

a. probability **b. Area** c. cumulative probability d. none of these

x. In usual symbol, standard error of sample mean in SRSWOR is

a) $\frac{s}{\sqrt{n}}$ b) $\frac{s}{\sqrt{N}}\sqrt{\frac{N-n}{N-1}}$ c) $\frac{s}{\sqrt{N}}\sqrt{\frac{N-1}{N-n}}$ **d.none of these**

Q2.. Answer the following questions. **1X8**

i. If a Binomial distribution has mode X = 3 and 4, then find C.V. of the distribution.

Ans. n=8 and $p=\frac{1}{2}$. C.V.$=\frac{\sqrt{8*\frac{1}{4}}}{8*\frac{1}{2}}=\frac{\sqrt{2}}{4}$.

ii. Define stratified sampling?

Ans. In case of heterogeneous population the whole population to be divided into several homogeneous strata and units are drawn from it using SRSWOR.

iii. What do you mean by null hypothesis?

Ans. The statement stating the basic fact and which has to be tested.

iv. What is meant by a chance causes in statistical quality control?

Ans. Which can not be completely eliminated.

v. What is fpc ?

Ans. Finite population correction = $\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$.

vi. Write the moment estimator of m in Poisson (m).

Ans. sample mean.

vii. State Chebyshev's inequality.

Ans. $P(\mu-3\sigma < T < \mu+3\sigma) > \frac{8}{9}$

viii. Show that unbiasedness of the estimator $\hat{\theta}$ of the parameter θ does not necessarily imply that $\sqrt{\hat{\theta}}$ will be an unbiased estimator of $\sqrt{\theta}$,

Ans. $V(\sqrt{\hat{\theta}}) > 0$