Indranil Ghosh
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FOR GOD AND COUNTRY

ST. LAWRENCE HIGH SCHOOL

1^{st} Term Examination – 2019

MODEL ANSWERS

Sub: Mathematics | Class: 11A2,B,C,D | F. M. : 80

Duration: 3 hrs 15 mins | Date: 8.8.19

Group – A

1. **Choose the correct option for the following questions** **(8x1=8)**

i) The angle between the lines x=a and y=b is

a) 0^0 b) 90^0 c) 180^0 d) none of these

Sol:b)

ii) The centroid of the triangle formed by the points (1,2),(2,4),(-3,6)

a) (0.0) b)(0,4) c)(4,0) d) (4,4)

Sol:a)

Iii) If x,2x+1,14 are in A.P, then the value of x is

a) 2 b) 5 c) 10 d) 15

Sol:4

iv) If G.M of two numbers is ± 12,if one number is 16,then the other number is

a) 3 b) 6 c) 8 d) 9

Sol:d)

v) If 2+3i be a root of a quadratic equation,then another root is

a) -2-3i b)2-3i c) -2+3i d) 5i

Sol:b)

vi) The value of $1 + i + i^2 + i^3 + i^4$ is

a)0 b)1 c)i d)2

Sol:b)

vii) sin(A+B)sin(A-B)=

a)$\sin^2A-\sin^2B$ b)$\cos^2A-\sin^2B$ c)$\cos^2B-\sin^2B$ d)$\cos^2A-\cos^2B$

Sol:a)

viii)If $\sin\theta$=3/5,then value of $\cos2\theta$

a) 7/15 b) 8/25 c)2/5 d)7/25

Sol:d)

Group -B

2.Answer any six questions **$(6\times 4 = 24)$**

1. If cosA+cosB=2,find the value of cos(A+B)

Sol:1-cosA +1-cosB=0, $\sin^2A/2+\cos^2A/2=0$,A=0,B=0, so cos(A+B)=1

2. Prove $\tan70^0=2\tan50^0+\tan20^0$

Sol:$\tan70^0=\tan(50^0+20^0)=[(\tan50^0+\tan20^0)/1-\tan50^0\tan20^0)]$

$\tan70^0 - \tan70^0\tan50^0\tan20^0=\tan50^0+\tan20^0$ now ,$\tan70^0=\cot20^0=1/\tan20^0$

3. Find the value of $\cos20^0\cos40^0\cos80^0$

Sol:$\frac{1}{2}(2cos20°cos40°)cos80° = \frac{1}{2}(cos60° + cos20°)cos80° = \frac{1}{4}cos80° + \frac{1}{4}(2cos20°cos80°)$.again apply formula.Ans:1/8

4. Find the maximum value of $3\cos\theta+4\sin\theta+5$

Sol:let rcost=3,rsint=4,so r=5,tant=4/3.Ans.10 Page 136 textbook

5. Find the value of $\cot660^0+\tan(-1050^0)$

Sol:cot(8.90^0-60^0)=-cot60^0.Similarly find tan, Ans:0

6. If sin2A=4/5,find the value of sinA($0< A < \pi/4$)

Sol:Find tanA,from sin2A=2tanA/(1+$\tan^2$A),then sinA=$\frac{1}{\sqrt{5}}$

7. Find the value of 2cos$\frac{\pi}{8}$

Sol:2$\cos^2$A/2=1+cosA.Put A=45^0.Ans:$\sqrt{2+\sqrt{2}}$

8. Prove secα +tanα=tan($\frac{\pi}{4}$+$\frac{\alpha}{2}$)

Sol:Change sec and tan in cos and sin, Refer :page 170,example 9

Group C

3. Answer any six questions **($6\times 4 = 24$)**

1. State De Morgan's laws of sets.Prove any one by venn diagram.

Sol: $(A\cup B)^C=A^C\cap B^C$ $(A\cap B)^C=A^C\cup B^C$.Refer Page 14 for figure

2. If a,b,c,d are in G.P,show that $(a-b)^2,(b-c)^2,(c-d)^2$ are in G.P

Sol:$\frac{b}{a}=\frac{c}{b}=\frac{d}{c}=k$, b=ak,c=bk=$ak^2$,d=ck=$ak^3$,R.T.P $[(b-c)^2]^2=(a-b)^2(c-d)^2$

3. If ω be an imaginary cube rootof unity,find the value of $(3+5\omega^2+3\omega)^6$

Sol:3+3$\omega = -3\omega^2$ Ans:64

4.Solve:$3x^2-(2-i)x+10-4i=0$

Sol:Use Sridhar Acharya's formula.Page328,5iii.Ans:x=-2i,(2+5i)/3

5.Find the sum to n terms 1+3+6+10+15+.....

Sol:S=1+3+6+10+... (A) S=0+1+3+6+...+t_{n-1}+t_n (B). Subtracting(A)-(B)

T_n=(1+2+3+...upto n terms)=n(n+1)/2 ,S=$\sum n(n+1)/2$ =n(n+1)(n+2)/6

6.If 51+53+55+...+t_n=5151,find the value of t_n

Sol:$5151 = \frac{n}{2}[2 \times 51 + (n-1) \times 2]$solving n=51,$t_n$=51+(51-1)$\times$ 2 =151

7.Solve:|2x-3|≤ 1 and |5-2x|≥ 3

Sol:Take +ve & -ve sign $\pm$(2x-3)≤ 1 ,$\pm(5-2x) \geq 3$,Solve.Ans:[1,2],($-\infty, 1] \cup [4, \infty)$

8.If |2z-1|=|z-2|,where z=x+iy,prove that $x^2+y^2=1$

Sol:$\sqrt{(2x-1)^2+(2y)^2}$=$\sqrt{(x-2)^2+y^2}$.Simplify

Group D

4.Answer any six questions **($6\times 4 = 24$)**

1. Find the image of the point (5,-7) with respect to the straight line 2x-3y=18

Sol:Ans(1,-1)Page 552,example 4 of textbook

2. Show that the four straight lines x+2y+5=0,2x+y+1=0.2x+y+7=0,x+2y-1=0 form a rhombus

Sol:Solve the equations to get vertices.Then find length of sides which will be equal.Sides are not perpendicular.Refer page544 example20

3. The perimeter of the triangle formed by the straight line 4x+3y-k=0,with the coordinate axes is 24 units,find the value of k

Sol:intercepts on x and y axis are k/4,k/3.length of other side $(k^2/16+k^2/9)^{1/2}$=5k/12.so k/3+k/4+5k/12=24, Ans:k=24

4. If 3a+2b+c=0,show that the equation ax+by+c=0 always passes through a fixed point.

Sol:c=-3a-2b put the value of c,the straight line becomes a(x-3)+b(y-2)=0. Clearly point of intersection is x=3,y=2 which is the fixed point

5. For what value of a,the three straight lines 7x-11y+3=0,4x+3y-9=0 and 13x+ay-48=0 are concurrent

Sol:Solve the first two equations,put the value of x,y in third equation to get a=26

6. Find the area of the triangle having vertices (a,b+c),(b,c+a),(c,a+b) and interpret the result geometrically

Sol:Use area of triangle formula, area=0.points are collinear

7. The coordinates of a moving point are (ct+c/t,ct-c/t),where t is a variable parameter.Find the equation to the locus of P

Sol:h=ct+c/t,k=ct-c/t, $h^2-k^2=(ct+c/t)^2-(ct-c/t)^2=4c^2$

8.Show that the product of the perpendiculars drawn from the two points $(\pm\sqrt{a^2-b^2},0)$ upon the straight line $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$ is b^2

Sol:$p=\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}$ .Use this formula. Here put $(x_1,y_1)=\pm\sqrt{a^2-b^2},0)$

a$=\frac{\cos\theta}{a}, b=\frac{\sin\theta}{b}$,c=-1, $p_1=\frac{b\sqrt{a^2-b^2}\cos\theta-ab}{\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}}$.In p_2 it will be +ab.Find p_1 p_2.take mod